Basic Concepts of Atonality

What follows is a series of definitions of terms which are basic to understanding atonal music (*any* music, I would say, although some of these ideas have less relevance to tonal music).

1. A **pitch** is a tone in the 12-tone equal-tempered scale. We could limit ourselves to the 88 pitches available on the piano, which span seven octaves and a minor third, but in fact we can hear a bit below and above these limits. Rarely, however, do we hear sounds in music that exceed this gamut. Most musical compositions use a smaller range than that, and indeed the bulk of "Western" music could be restricted to a range of five or six octaves. Each pitch can be represented in several different ways in Western musical notation.

Pitch is, of course, a basic concept in all music, and I do not wish to overlook the many complications associated with the subjects of tuning, microtonality, or on a more basic level, the fact that tones with pitches constitute only a subset of sounds available. Pitch is a quality that we hear only on tones that have harmonic partials. These subtleties are things that need to be explored in more detail, but not in this context.

2. A **pitch class,** abbreviated **PC,** is the set of all notes which differ from one another by one or more octaves. Another way of stating this is to say that all members of a pitch class share the property of **octave equivalence,** by which we recognize their similarity. Still another is to say that pitch classes include all the notes C, C-sharp or D-flat, etc. There are 12 pitch classes in the 12-tone equal-tempered scale.

3. A **pitch class collection,** abbreviated **PC coll,** is a set of distinct pitch classes. There are 4096 (2^{12}) PC colls. Important properties of a PC coll is its **size,** usually denoted by **n,** and its structure.

4. **Names of sets:** PC colls of two elements are called **dyads** or **intervals**. Sets of 3 through 8 elements are called **trichords, tetrachords, pentachords, hexachords, septachords,** and **octachords.** Larger sets call also be called "8-colls," "9-colls," etc. It is far easier to refer to sets larger than hexachords by **excluded notation,** indicating the notes excluded by the collection. I will usually employ this notation.

5. A **subcollection** of a PC coll is a subset of the PCs in the collection. Two subcollections which, between them, include all the notes of the PC coll without duplications, are said to be **complementary.**

6. A **transposition** (**Tn**) of a PC coll (or pitch or PC) is the addition of a number of semitones to each element of the PC coll, thus moving it up or down by a number of semitones. There are a maximum of 12 distinct transpositions of a PC coll. A PC coll (or PS) that has fewer than 12 distinct transpositions is said to be **degenerate** (see below).

7. An **interval** is the difference, in semitones, between two pitch classes. There are 12 intervals. Two intervals are said to be **complementary** if their sum, modulo 12, is zero, meaning that they add up to an octave.

8. **Interval class** is a term denoting pairs of complementary intervals, or pitch structures of size 2 (see immediately below). There are six interval classes. Interval class is a useful term

when we want to describe, for example, major thirds and minor sixths as a single concept.

9. A **pitch structure,** abbreviated **PS,** is a set of PC colls related by transposition, and only transposition. It is my assumption that each individual pitch structure has its own distinct quality, such as "major triad-ness" for the major triad. The complete list of PSs is shown in appendices 1 and 2.

10. The **interval content of a pitch structure** is the number of interval classes of each contained in the PS. The standard way of notating the interval content is as a succession of six integers enclosed in square brackets, denoting the number of instances of interval 1, 2, ..., 6. An example is [221100] for the PS 0124.

11. **Integer notation for notes, PC colls, and PSs:** Because musical notation has numerous ways of writing out each pitch (very important for showing the structure of tonal music!), and because it gets very cumbersome to try to figure all this out on the musical staff, a system of **integer notation** has been developed. I use exclusively the **"fixed do"** system, where 0 indicates C, 1 C# or D-flat, 2 D, ..., 11 B. Thus, instead of writing C-D-E-F, you would instead write 0245. The same type of process is used when indicating a pitch structure, except in this case the numbers indicate the difference, in semitones, from the lowest element. Thus, 0245 would indicate that the second, third and fourth elements of the set were 2, 4 and 5 semitones above the lowest pitch. Because the numbers 10 and 11 have two digits, some authors indicate these as "T" and "E". This convention is not followed here, partly because both of those letters have other meanings, and for PSs, 10 only occurs in one case, for the "whole-tone scale" 0246810.

12. **Normal form notation for pitch structures:** Each pitch structure (greater than 1 and less than 11) has several different registral representations, equal to the number of elements in the set: trichords have three (familiar from the inversions of triads), tetrachords four, etc. Each of these representations should be written out in ascending order before making the choice of which to use as the notation of the PS. For example, the major triad could be written out as 047, 038, or 059. (These are determined by subtracting the lower note from the higher ones, modulo 12.) In order to have a consistent notation for PSs, it is necessary to choose one of these orderings for the notation of the PS. That notation is called the **normal form.** The normal form definition states that,

(1) if one representation has the smallest overall span, that is chosen as the notation for the PS.

(2) If two or more representations have the same overall span, then the choice is the one with the smallest 2nd, 3rd, etc. interval. (For example, 027 could be written as 027, 0510, or 057. 027 and 057 have the same overall span, but 027 has a smaller second element.)

13. **Notation for PC colls:** A PC coll is indicated as a transposition of a PS in normal form; the transposition number is written as a subscript after the PS. For example, the B-flat major triad would be indicated as 047_{10}. In this case, the transpositions are always indicated according to the fixed-do system. The pentatonic scale or "black key" collection could be written as 02479_6 (note that the normal form starts on F#). For PSs larger than hexachords, while PS notation exists for these sets, such as 01356810_{11} for the "white key" collection or the C-major scale (normal form starts on B), it is easier to refer to these sets by the notes that they

exclude or their complements. Thus, the major scale collection would be written as **Excl. 02479_6**.

14. **Modulo 12 arithmetic:** Because of octave equivalence, when a PC is transposed beyond 12, it needs to be reduced to the same octave. This means that 12 is subtracted from (or added to) the number obtained repeatedly until the value lies within the range 0 to 11.

15. **Degenerate Pitch Structures:** A degenerate PS is one that has less than 12 distinct transpositions. For example, the "diminished seventh chord" 0369 maps into the same set of notes at transposition of 0, 3, 6 and 9. The PC coll could be referred to as $0369_{0,3,6,9}$, but usually only the PC that is relevant in a particular context will be used (such as 0369_6 in a context where 6 has some special property).

16. **Common Tone Relation (CTR):** The CTR denotes the number of PCs in common between two PC colls. For example 047_0 and 02479_0 contain 3 tones in common, and in fact 047_0 is a subcollection of 02479_0. 037_4 and 02479_0 have two tones in common.

17. A **multiplicative operation, Mn,** is an operation that maps the PCs in a PC coll or PS into different PC coll or PS, obtained by multiplying the integers indicating the PC by an integer modulo 12. While it is possible to use any integer for a multiplicative operation, only a few of them are useful, because some of them map different PCs into octaves of other PCs, thus failing to preserve the unique size of the set. (For example, M4 maps the PCs of the PS 013 into 0, 4, and 0 (12 modulo 12), whereas M5 maps it into 0, 3 and 5.) There are four multiplicative operations that are basic to the system described herein, as follows:

M1 identity (abbreviated ID)
M11 inversion (abbreviated INV)
M5 cycle-of-fourths equivalence (abbreviated 5C INV)
M7 cycle-of-fifths equivalence (abbreviated 5C)

Cycle-of-fourths and cycle-of-fifths equivalences are twentieth-century terms. Inversion, on the other hand, has been widely used throughout music history.

18. A **set class** is a set of pitch structures containing both the prime and inversional forms, if the inversion exists as a separate PS. Like interval class, set class is useful when we want to describe the properties of a set and its inversion as a single entity, such as "major-minor triad-ness" as opposed to "major triad-ness" or "minor triad-ness".

19. **Type classification of PSs:** Type classification describes the behavior of PSs under multiplicative operations. They are as follows:

Type 0: the PS is degenerate (has less than 12 distinct transpositions)
Type 1: the PS yields the same structure under M1, M5, M7 and M11.
Type 2: the PS yields the same structure under M1 and M11 and a different structure under M5 and M7.
Type 3: the PS yields the same structure under M1 and M7 and a different structure under M5 and M11.
Type 4: the PS yields the same structure under M1 and M5 and a different structure under M7 and M11.

Type 5: the PS yields different structures under each of the operations M1, M5, M7 and M11.

20. **Pitch Structures: the Data:** The list of all the pitch structures of sizes 3 through 6 that exist according to the definitions above is shown in a separate document. Since any collection larger than 6 can be indicated in exclusion notation, there is no need to know larger sizes. Let's comment on these data:

Trichords: There are 19 of them, including one type 0 degenerate (048, also called the **augmented triad**), and two sets of type 5 groups, the 013 group and 014 group, the latter of which includes the major and minor triads.

Tetrachords: There are 43, including three type 0 degenerates 0167 and 0268 (the "French" augmented sixth chord), which have 6 transpositions, and 0369 (the **diminished seventh chord**) which has only 3. There are five sets of type 5 groups, identified by the letters A, B, C, D and E. Of these, the D group is the most interesting, because each of these possess all intervals, one instance of each. The E group includes the 0368 (the **dominant seventh chord**) and its inversion 0258 (the **half-diminished seventh or "Tristan" chord**). In using the group letter designations, I will usually also indicate the structure of the first tetrachord in parentheses (i.e., group A (0124) tetrachords), and similarly for pentachords and hexachords.

Pentachords: Since 5 does not divide into 12 evenly, there are no type 0 degenerate sets, and there are 10 type 5 groups, identified by the letters A through J.

Hexachords: There are 80 of them, including three type 0 degenerates 012678, with 6 transpositions, 014589 (two consecutive augmented triads), with 4 transpositions, and 0246810, the **whole-tone scale**, which has only two distinct transpositions. As with pentachords, there are 10 type 5 groups, identified by the letters A through J. Special note must be made of the unique type 3 hexachords 013679 and 023689. These yield different PSs under M11, but each has only six distinct transpositions.

21. **Partition:** the division of a larger set into smaller sets, such that all PCs are used. A 12-tone set, for example, can be partitioned into two hexachords, three tetrachords or four trichords. An octachord can be partitioned into two tetrachords or one hexachord and one dyad.

22. **Complement:** The set of PCs excluded by a given set. It is also possible to speak of the complement to an octachord or any another smaller set.

23. **Self-inverting:** a term applied to a PS that yields itself under inversion.

24. **Total chromatic:** The set of all 12 PCs. In 12-tone music, this is also referred to as an **aggregate,** usually in contexts where ordering is not under consideration.

25. **Z-Relation:** A relationship that exists when two PSs have the same interval content but are not transpositions or inversions of one another. (For example, the "all-interval tetrachords" 0146 and 0467 each have the interval content [111111].) While PSs related by inversion have the same interval content, the same interval content also exists for other sets. Those sets are related by multiplicative operations, however (0146 and 0467 are related by M7). The Z-relation is a concept that was discovered by theorists who took the interval content of a set

as the basic concept with which to explore atonal music but then this didn't always work.

26. **Developing variation:** A concept used by Schoenberg and his students which came from late nineteenth-century romanticism but which found application in early atonal music. Rhythmic motives and interval patterns are used with increasing variation on successive returns, bringing in newer ideas that gradually produce new motives.